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INTERNAL LETTER

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Subject: TCXO Thermistor Resistance Calculations

1. Theory

Several models have been developed to describe the resistance-temperature characterisitics of thermistors. The simplest model I know is the Beta Equation. This will give reasonable accuracy over limited temperature ranges.

Beta Equation

$$R(T_n) = R(T_o)\,e^{\beta_o\left[\frac{1}{T_n} - \frac{1}{T_o}\right]} \quad \text{ohms}$$

The parameters for the Beta Equation are defined as follows:

T_n = Thermistor body temperature in degrees Kelvin ($25^oC = 298.15^oK$).

T_o = Temperature at which nominal thermistor resistance, $R(T_o)$, is determined. A reasonable choice is $T_o = 298.15^oK$ (25^oC).

$R(T_o)$ = Thermistor resistance at a body temperature of T_o.

$R(T_n)$ = Thermistor resistance at a body temperature of T_n.

β_o = Material constant.

This simple "Beta" model is based on the assumption that the resistance varies inverserly with temperature in a "perfect" logrithmic fashion. This can be demonstrated by writing the Beta Equation in its logarithmic form:

$$\ln\left\{\frac{R(T_n)}{R(T_o)}\right\} = \beta_o\left[\frac{1}{T_n} - \frac{1}{T_o}\right]$$

When this logarithmic equation is plotted against an actual thermistor curve, the value of β_o turns out to be slightly temperature dependent. This can be accounted for by trying to "curve fit" β with a polynomial in the Modified Beta Equation.

Modified Beta Equation

If we represent the slight temperature dependence of β by a polynomial, the Beta Equation is modified to give:

$$\beta(T_n) = B_o + B_1(T_n - T_o) + B_2(T_n - T_o)^2 + B_3(T_n - T_o)^3$$

$$R(T_n) = R(T_o)e^{\beta(T_n)\left[\frac{1}{T_n} - \frac{1}{T_o}\right]}$$

The parameters B_o, B_1, B_2, and B_3 are now material constants to be determined by curve-fitting the logarithmic equation:

$$\beta(T_n) = \left[\frac{1}{T_n} - \frac{1}{T_o}\right]^{-1} \ln\left\{\frac{R(T_n)}{R(T_o)}\right\}$$

To determine the best least-mean-square error for the B parameters, $\beta(T_n)$ is determined at four spaced temperature values and the following matrix operation performed:

$$\underline{B} = [T]^{-1}\,\underline{D}$$

where:

$$\underline{B} = \begin{bmatrix} B_o \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} \quad \underline{D} = \begin{bmatrix} \beta(T_1) \\ \beta(T_2) \\ \beta(T_3) \\ \beta(T_4) \end{bmatrix} \quad T = \begin{bmatrix} 1 & (T_1-T_o) & (T_1-T_o)^2 & (T_1-T_o)^3 \\ 1 & (T_2-T_o) & (T_2-T_o)^2 & (T_2-T_o)^3 \\ 1 & (T_3-T_o) & (T_3-T_o)^2 & (T_3-T_o)^3 \\ 1 & (T_4-T_o) & (T_4-T_o)^2 & (T_4-T_o)^3 \end{bmatrix}$$

Notice for this model that if β is a constant, then $\beta_o = B_o$ and the simple Beta model applies.

Western Thermistor Equation

A third model has been developed by Western Thermistor to be used in calculating thermistor values. Their model actually uses four constants that will be designated A, B, C, D. Material constants A, B, and C are determined by experimental measurements. The constant D is determined by the range into which the nominal value falls. In some of their literature, D and R_{25} are used interchangeably which results in some confusion. The logarithmic form of the equation may be written as:

$$\frac{1}{T_n} = A + B \cdot \ln\left\{\frac{R(T_n)}{R(T_o)}\,D\right\} + C \cdot \ln^3\left\{\frac{R(T_n)}{R(T_o)}\,D\right\}$$

or alternately,

$$\frac{1}{T_n} = A + B \cdot \ln \alpha + C(\ln \alpha)^3$$

with:

$$\alpha = \frac{R(T_n)}{R(T_o)} D$$

If a nominal thermistor value is chosen such that

$$R(T_o) = D = R_{25}$$

then the equation simplifies to the one published in Western Thermistor literature:

$$\frac{1}{T_n} = A + B\ (\ln R(T_n)) + C(\ln R(T_n))^3$$

The Western thermistor equation is a cubic with no second order term and has a closed form solution. Appendix I shows a simplified form of the solution for thermistor resistance called the Interpolation Equation.

2. Application

For Western Thermistors, each thermistor is characterized by a curve type number. From a Table of R_{25} values, a device is selected and a resulting curve number obtained. Corresponding to a curve number, the parameters A, B, C, and D are determined. The temperature curve or resistance interpolation curve may then be computed.

Example

A 20K ohm thermistor characterized by curve 7 is chosen. From the table of parameters we obtain:

$A = 9.9790127 \times 10^{-4}$
$B = 1.9087826 \times 10^{-4}$
$C = 1.0379321 \times 10^{-8}$
$D = 100000\Omega$

Since we have chosen $R(T_o)$ = 20 KOHMS, the alpha factor is;

$$\alpha = \frac{R(T_n)}{2x10^4} \cdot 10^5 = 5R(T_n)$$

or conversely

$$R(T_n) = \frac{1}{5}\alpha$$

Using the program in Appendix III, the resistance at three temperatures is calculated by the interpolation equation and compared to the table:

T_n (°C)	$R(T_n)$	$R(T_n)/R(T_o)$	RT/R_{25}
-30	446.7K	22.34	22.35
+25	20.01K	1.001	1.000
+70	2.847K	0.1424	0.1426

Likewise, using the program in Appendix II, the temperature is calculated for three measured resistance values and compared to the table:

$R(T_n)$	T_n (°C) Calc.	T_n (°C) Table
236.0K	-20.00	-20
20K	+25.01	+25
6.420K	+49.99	+40

3. <u>Accuracy of the Model</u>

Tables of thermistor values have been calculated using the equations but their accuracy from batch to batch may vary widely. In addition, there is the nominal tolerance of the device computed at 25°C or 70°C.

Variations in actual thermistor performance compared to the interpolation equation can be considerable. For example, a curve 12 thermistor which is normalized to 70°C can have as much as ± 13.7% error at -20°C when compared to the value predicted by the equation. Efforts to verify the equations given in Appendix IV have failed. The constant, C, cannot be verified using the values published in the Table. I don't feel it justifies any more time. If the use of these equations becomes important, I'll contact Western and work out the details.

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Appendix I. Simplification of the Western Thermistor Interpolation Equation

The interpolation equation published by Western Thermistor is:

NTC THERMISTOR RESISTANCE VERSUS TEMPERATURE INTERPOLATION EQUATION

$$\ln R_{T_1} = \frac{1}{3}\left[\sqrt[3]{-\frac{27}{2}\left(\frac{A-T_1^{-1}}{C}\right) + \frac{3}{2}\sqrt{3}\sqrt{27\left(\frac{A-T_1^{-1}}{C}\right)^2 + 4\left(\frac{B}{C}\right)^3}} - \sqrt[3]{+\frac{27}{2}\left(\frac{A-T_1^{-1}}{C}\right) + \frac{3}{2}\sqrt{3}\sqrt{27\left(\frac{A-T_1^{-1}}{C}\right)^2 + 4\left(\frac{B}{C}\right)^3}}\right]$$

First we note that the Thermistor resistance term should really be α where:

$$\alpha = \frac{R(T_n)}{R(T_o)} D$$

Using the following substitutions,

$$\theta_1 = \frac{1}{C}\left(A - \frac{1}{T_n}\right)$$

$$\theta_2 = \sqrt{\theta_1^2 + 4\left(\frac{B}{3C}\right)^3}$$

The interpolation equation is written as:

$$\ln\{\alpha\} = \frac{1}{\sqrt[3]{2}}\left[\sqrt[3]{-\theta_1 + \theta_2} - \sqrt[3]{+\theta_1 + \theta_2}\right]$$

Appendix II. HP-25 Program for Calculating Temperature from $R(T_n)$.

Memory	
STO7	A
STO6	B
STO5	C
STO4	D
STO3	$R(T_o)$
STO2	273.15^oK
STO1	ln α

Enter A, B, C, D, $R(T_o)$, T_o

Enter $R(T_n)$ ohms

01	↑	16	x
	RCL3		+
	÷		RCL7
	RCL4		+
05	x	20	g 1/x
	f ln		↑
	STO 1		↑
	↑		RCL2
	↑		-
10	x	25	GTO00
	x		
	RCL5		
	x		
	RCL1		
15	RCL6		

X	Y
T_n (°C)	T_n (°K)

Example: (Curve 7)

$A = 9.9790127 \times 10^{-4}$
$B = 1.9087826 \times 10^{-4}$
$C = 1.0379321 \times 10^{-8}$
D = 100,000 ohms
$R(T_o) = 2 \times 10^4$ ohms
$T_o = 273.15$

$R(T_n) = 7.28 \times 10^4$

$T_n = 0.073^oC$

Appendix III. HP-25 Program for Thermistor Resistance Calculated from T_n

Enter A, B, C, D, $R(T_o)$, T_o

Enter T_n ^oC

```
01   RCLO
      +
     g 1/x
     CHS
05   RCL7
      +
     RCL5
      ÷
     STO 1
10   g x²
     RCL6
     RCL5
      ÷
      3
15    ÷
      ↑
      ↑
      X
      X
20    4
      X
      +
     f √x
     STO2
25   RCL 1
26    -
      3
     g 1/x
     f y^x
30   RCL1
     RCL2
      +
      3
     g 1/x
35   f y^x
      -
      2
      ↑
      3
40   g 1/x
     f y^x
      ÷               X/α
     ge^x
     RCL3
45    X
     RCL4
      ÷
48   GTO00            X/R(Tn) ohms
```

Memory	
STO7	A
STO6	B
STO5	C
STO4	D
STO3	$R(T_o)$
STO2	θ_2
STO1	θ_1
STO0	273.15^oK

Appendix IV Calculation of Material Constants from Measured Data

Western Thermistor provides the following equations for calculating A, B, and C from measured data:

$$C = \frac{x_3(y_1-y_2) - x_1(y_1-y_2) + y_1x_1 - y_1x_2 - y_3x_1 + y_3x_2}{(y_3)^3(y_1-y_2) - (y_1)(y_2)^3 + (y_1)^4 - (y_1)^3(y_1-y_2) + (y_3)(y_2)^3 - (y_3)(y_1)^3}$$

$$B = \frac{x_1 - x_2 + C(y_2)^3 - C(y_1)^3}{(y_1-y_2)}$$

$$A = x_2 - By_2 - C(y_2)^3$$

where:

$$x_1 = \frac{1}{T_1} \qquad y_1 = \ln \alpha_1 \qquad \alpha_1 = \frac{R(T_1)}{R(T_o)} \; D$$

$$x_2 = \frac{1}{T_2} \qquad y_2 = \ln \alpha_2 \qquad \alpha_2 = \frac{R(T_2)}{R(T_o)} \; D$$

$$x_3 = \frac{1}{T_3} \qquad y_3 = \ln \alpha_3 \qquad \alpha_3 = \frac{R(T_3)}{R(T_o)} \; D$$

All temperatures in $^{\circ}K$ $\qquad 0^{\circ}C = 273.15^{\circ}K$

D is determined by Curve number

$$-40^{\circ}C \leq T_1 \leq T_2 \leq T_3 \leq 150^{\circ}C$$

$$(T_2 - T_1) \leq 50^{\circ}C \quad \text{and} \quad (T_3 - T_2) \leq 50^{\circ}C$$